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Heat transfer in laminar flow in a concentric annulus with peripherally varying heat transfer

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Abstract

A theoretical analysis of heat transfer in fully-developed laminar flow in a concentric annulus with peripherally varying and axially constant heat flux has been made. To determine the temperature, superposition of the temperature solutions for the average axisymmetric heat-flux component and the perturbation heat-flux component (the boundary integral of which is zero) have been used. The axisymmetric temperature solution is already known, and the perturbation temperature solution has been obtained numerically using a point-matching method. A number of examples are presented to demonstrate the method, and to show typical wall temperature and heat transfer coefficient distributions around the boundary of the duct.

The use of the average heat transfer coefficient in the case of peripherally varying boundary conditions has been critically examined. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

A cross-sectional area
c specific heat of fluid
b_o, a_0, k_n, c_n, d_n constants
d diameter, or equivalent diameter of duct
h heat transfer coefficient
k thermal conductivity
Nu $(=hd/k)$ Nusselt number
n integer
q, $\bar{q}$ heat flux, average heat flux
r radius
T, $\hat{T}$ temperature
u, U velocity, average velocity.

Greek symbols
α, θ angle
Δ perturbation or difference
ρ density.

Subscripts
B bulk
m value at maximum velocity
p perturbation heat flux
u uniform heat flux
w wall
1 inner radius
0 outer radius.

1. Introduction

Convection heat transfer to fluid flow in long straight ducts of constant cross-section is of special interest to mechanical and chemical engineers, and considerable effort has been devoted over the past 60 years or so to its understanding and prediction. With the advent of high speed computational facilities and sophistication in mathematical techniques and experimental methods, numerous problems in this area of study can now be solved.

Of course most realistically, the heat transfer to the flow cannot be taken in isolation. That is, the heat trans-

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fer to the flow takes place conjugately and interdependently with the energy transfer in the surroundings, and the thermal conditions at the boundary of the domain of interest, i.e. the ducted flow, are adjusted accordingly. The temperatures and heat fluxes at all the points of the boundary are then fixed so as to maintain the energy balance between the adjacent regions. In this connection, many of the studies of convection heat transfer considered the more simple idealised thermal boundary conditions, those of constant temperature and constant heat flux being the most familiar. These two cases in particular have been used to demonstrate the dependence of heat transfer performance on thermal boundary conditions, and as an approximation to actual conditions. However, numerous more realistic thermal boundary conditions are readily brought to mind, and it is immediately apparent that variations of both temperature and heat flux may occur around the perimeter of the cross-section as well as in the flow direction. Peripheral variations will be considerable in the case of cross-flow heat exchangers, in the radiant water-tube banks of boilers, and in the absorber tubes of many types of solar collector, to cite but three examples. Of all the boundary conditions considered to date, the general cases of arbitrary peripheral variations of temperature or heat flux are the least well-known, and their analyses are very limited. Indeed in most heat exchanger applications, some temperature or flux variations around the boundary is likely to occur, and so its influence on heat transfer performance and other thermal parameters is important.

While the problems associated with peripheral variation have been recognised as long ago as 1952 in connection with annular flow [1, 2] it appears that the first quantitative assessment of the effect of the peripheral variation of heat flux in the case of a tube was made by Reynolds about 1960. In his work Reynolds [3, 4] showed that there could be a pronounced influence of the circumferential heat flux variation in both laminar and turbulent flows. While reference had been made to the corresponding annulus problem [5], it appears that equivalent studies of peripherally varying boundary conditions was not made until 1964, albeit that most other cases for this geometry had been investigated in a systematic manner. At that time, Sutherland and Kays [6], extended the analysis of Reynolds [4], to the annulus. In the early 1960s, a very detailed theoretical and experimental investigation which was carried out over four years, was made into the annulus problem by workers at Stanford University [7–10]. In that study the annulus problem was formulated in a general way, and a number of 'fundamental solutions' (four in all) was obtained for the axisymmetric case. The possible extension to include the aforementioned circumferential variation was recognised then but the authors did not pursue this matter further at that time. One of the purposes of the present paper is to continue with the annulus study using a calculation procedure which has been developed by the authors in connection with an earlier heat conduction problem [11]. The method to be described may, in principle, be used for any constant cross-section duct with the proviso that the energy transfer to the flow is constant in the flow direction. Accordingly, a comparison can be made with the results obtained by Reynolds [3] for a round tube with arbitrary circumferentially varying heat flux.

In formulating the problem and in describing the solution procedure, it is more complete and convenient to use the concentric annulus geometry and this is done at the outset in the following section. The corresponding tube analysis (the form of which is identical with that of the annulus with appropriate changes), is presented later and the comparison with the previous work of Reynolds [3] is made. A further feature of the present investigation is that advantage is taken of the existing results of the pertinent fundamental solution for the annulus having no peripheral variation of the heat flux. Finally, typical numerical examples for both the tube and the annulus are included with attention being focused on the effect of peripheral variation of heat flux on wall temperature and local heat transfer coefficient. With regard to heat transfer coefficient, the meaning of 'average heat transfer coefficient' in the present situation is clarified. In the next section, the fundamental principles and the calculation method are outlined.

2. The solution procedure

As mentioned in the previous section, the concentric annulus has been chosen for the purpose of explaining the present method of solution of the peripherally varying heat flux problem. If the usual idealisations associated with a low speed (laminar) constant property fluid with no internal heat generation are assumed, then in general,

$$\nabla^2 T = \left(\frac{u\rho c}{k}\frac{\partial T}{\partial x}\right) \tag{1}$$

for steady hydrodynamically and thermally developed flow. Furthermore, for a constant axial heat flux, i.e. constant energy transfer to the flow in the flow-wise direction,

$$\frac{\partial T}{\partial x} = \frac{\partial T_{\mathrm{B}}}{\partial x}. \tag{2}$$

Now since the energy equation (1) is linear, then following [12], advantage may be taken of the superposition of simpler solutions with their appropriate boundary conditions. By subdividing the peripheral heat flux into an axisymmetric distribution and a perturbation component, the boundary integral of which is zero, the problem may be reduced to: (i) an axisymmetric laminar convection problem, the solution of which is readily available [12], and (ii) a pseudo-conduction problem, or laminar convection problem with zero axial bulk temperature gradient. The solution of this problem necessitates the solution of the Laplacian in the duct cross-section, since with equation (2) the right hand side of equation (1) is now zero. (With a specified perturbation of boundary heat flux, this is a Neumann problem.)

The general axisymmetric geometry and heat transfer solution is depicted in Fig. 1 where the basic equations and their corresponding general solutions are included for completeness. The final temperature solution is simply the sum of the temperature solutions for the axisymmetric and perturbation heat flux problems.

In engineering heat transfer, the convective heat transfer coefficient or Nusselt number is also of interest, and it is then necessary to determine the differences between the local wall temperature and the adjacent bulk (or mixed-mean) temperature in the flow. This information is at hand for the axisymmetric (i.e. uniform heat flux) case [12], while for the complementary perturbation heat flux problem, Fig. 1(c),

$$T_{pB} = \frac{\int_A uT\,dA}{\int_A u\,dA} = \frac{\int_A u\hat{T}\,dA}{\int_A u\,dA} + C_3 \tag{3}$$

must be evaluated once $\hat{T}$ has been determined as outlined later.

The superposition aspect of this problem may be demonstrated most effectively in the following way. An axisymmetric problem, for which the solution is already known, is chosen and this is then split into two simple axisymmetric problems in accord with the criteria set earlier. For convenience the following data are assumed:

(i) Inner and outer radii, 0.03 and 0.05 m, respectively;
(ii) Inner heat flux and outer heat flux, 200 and 100 W m^{-2}, respectively.

Figure 2 shows the situation and how the original problem is subdivided into system (i) corresponding to the 'uniform flux', and system (ii) corresponding to the 'perturbation' problem. Of course, in this simple illustration the perturbation is uniform around the boundary, and in order to conform to the zero net energy transfer criterion of the pseudo-conduction problem the fluxes in system (ii) are chosen to satisfy: $q_1r_1 = q_0r_0$. Now, for the three uniform heat flux cases shown in Fig. 2, the wall to bulk temperature differences may be determined from the 'fundamental solutions' (of the second kind) given in ref. [12]. The temperature differences are shown for each case, in Fig. 2, and it will be seen that the sum of the temperature differences in the two component systems equals the temperature difference in the original system, thereby verifying the calculation procedure. Of course, the original problem may be solved completely without any reference to superposition, but the example shows the general principles which need to be adopted in the more general peripherally varying heat flux case.

When the heat flux varies around the boundary of the duct, the procedure is to apportion the flux into its uniform and perturbation components as before. The uniform flux problem may be dealt with in the usual way using the fundamental solutions of the second kind, while the temperature associated with the perturbation requires separate consideration as outlined in the following paragraphs where a point-matching scheme [13] is employed.

Since the boundary integral of the perturbation heat flux is zero, then $\nabla^2 T = 0$, the solution for which is:

$$T = C_3 + b_0 \ln(r) + \sum_{n=1,2,\ldots} (a_n r^n + b_n r^{-n})\cos(n\theta) + \sum_{n=1,2,\ldots} (c_n r^n + d_n r^{-n})\sin(n\theta) \tag{4}$$

as reference to standard mathematical texts will reveal. When there is symmetry about PP then the sine terms in equation (4) vanish. Now in the present problem with specified heat flux at the annulus boundary, we have

$$\Delta q_1 = -k\left(\frac{\partial T}{\partial r}\right)_{r=r_1} \quad \text{and} \quad \Delta q_0 = +k\left(\frac{\partial T}{\partial r}\right)_{r=r_0} \tag{5}$$

and in order to effect the calculation of the coefficients a_n, b_n, c_n, and d_n for the general case, a numerical procedure is necessary. Following the procedure used by the present authors in an earlier study [11], equations (4) and (5) are used at a number of points on the boundary. The resulting set of linear equations may then be solved numerically with the result:

$$T = f(r,\theta) + C_3 = \hat{T} + C_3 \tag{6}$$

where C_3 is an arbitrary constant. As shown in the Appendix, for the very special case when there is symmetry about PP and, in addition, the perturbation heat flux is an odd function about VV the perturbation bulk temperature is simply C_3, and the local wall to bulk temperature difference is:

$$(T_w - T_{pB}) = (T_3 - C_3) = b_0 \ln(r_w) + \Sigma(a_n r_w^n + b_n r_w^{-n})\cos(n\theta). \tag{7}$$

The total local wall to bulk temperature difference is obtained by adding that according to equation (7) with that obtained from the fundamental solution of the uniform heat flux problem. The local Nusselt number is then calculated in the usual way.

Most generally the perturbation bulk temperature must be calculated from equation (3) with $\hat{T} = f(r,\theta)$ and:

$$u = 2U\left(\frac{r_0^2 - r^2 + 2r_m^2 \ln(r/r_0)}{r_0^2 + r_1^2 - 2r_m^2}\right) \tag{8}$$

with

$$r_m = \sqrt{\frac{r_0^2 - r_1^2}{2\ln(r_0/r_1)}} \tag{9}$$

for the laminar flow velocity field. However, as outlined

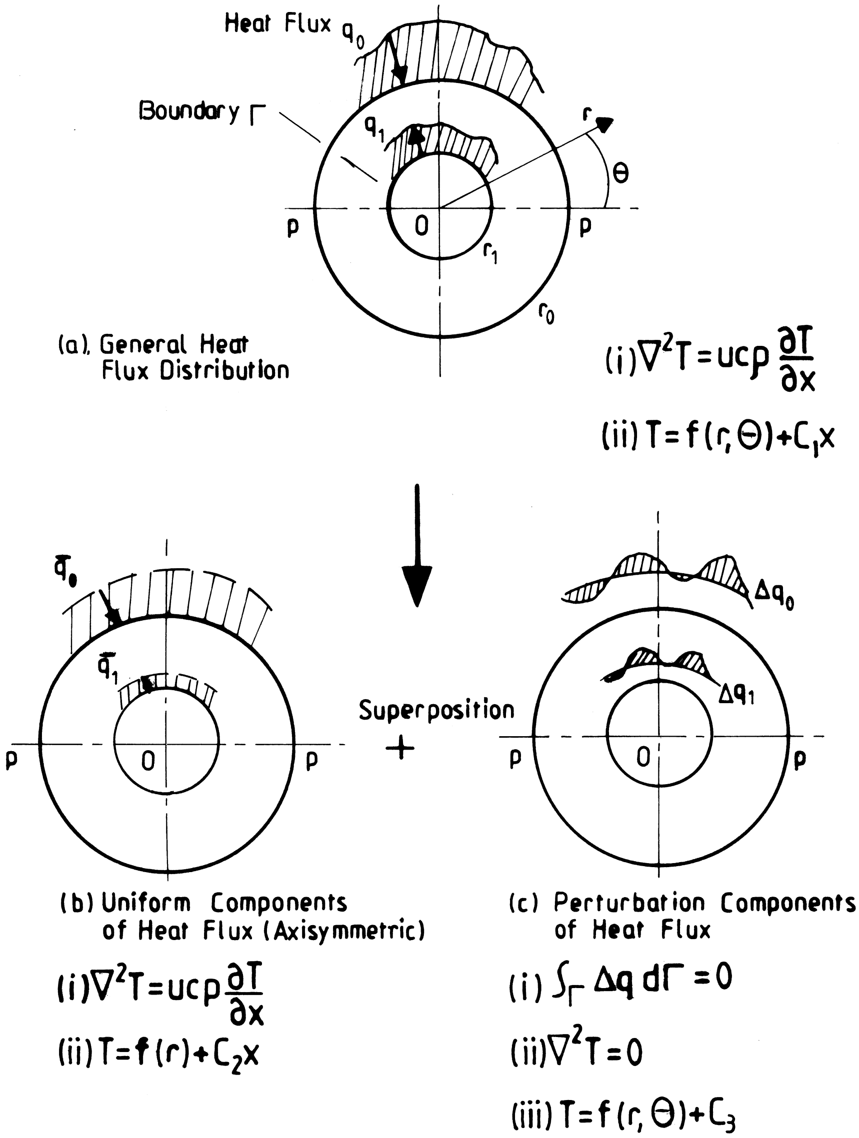


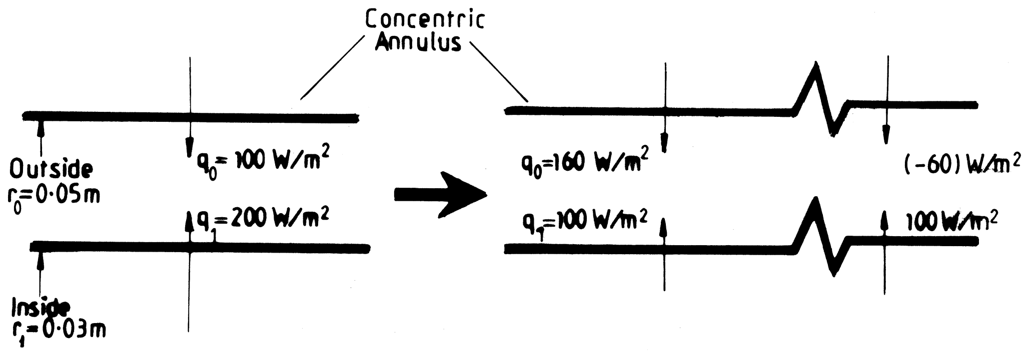
Fig. 1. Superposition principle applied to laminar convection heat transfer in an annulus with peripherally varying heat flux (q constant in x-direction).

in the Appendix, in the case of the annulus the numerical integration may be avoided and the perturbation bulk temperature obtained directly from the temperature solution.

The total local wall to bulk temperature difference is obtained again by summation. It is to be noticed that at no stage in this calculation is the actual temperature determined; this is of no consequence because it is the temperature difference which is of interest in assessing the peripheral variation of local wall temperature and local heat transfer coefficient.

3. The round tube with peripherally varying heat flux

In outlining the solution procedure for axisymmetric laminar flow with peripherally varying heat flux, the general axisymmetric geometry, i.e. the concentric annulus



			System (i)		System (ii)	
	1	2	3	4	5	6
	Nu	(T_W-T_B)	Nu	(T_W-T_B)	Nu	(T_W-T_B)
(a) Inner Surface	7·75	39·7°C	24·3	6·33°C	4·61	33·37°C
(b) Outer Surface	10·0	15·4°C	6·02	40·9°C	3·62	−25·5°C

(Air, k=0·026 W/m K)

Fig. 2. Example of superposition principle for laminar convection heat transfer in an annulus with axisymmetric heat transfer.

has been used. The calculation for the round tube is identical apart from the following modifications and simplifications:

In equations (4) and (7), b_0, b_n and d_n are inadmissible, Δq, the inner wall perturbation no longer exists, and equation (8) is replaced by:

$$u = 2U\left(1-\left(\frac{r}{r_0}\right)^2\right). \quad (10)$$

As for the annulus, the solution is obtained as the summation of the solutions for the uniform heat flux component and the perturbation heat flux component. Of course for the uniform heat flux $Nu = 4.36$ which leads to the corresponding wall to bulk temperature difference while the calculation of the local wall to bulk temperature difference due to the perturbation is effected using the simplified form of equation (4) in a point-matching scheme at points on the circumference. The procedure may be made more clear by means of a specific example as shown in Fig. 3(a). A cosinusoidal heat flux distribution has been chosen so that a direct comparison of the results may be made with those of Reynolds [3]. The disposition of the heat flux has been chosen intentionally so as to avoid symmetry about PP in order that the most general form of equation (4) may be used. In the present illustrative problem, 19 points over the range $(-\pi/2 \leqslant \theta \leqslant \pi/2)$ were selected.

The perturbation flux at each point was matched to the conduction heat flux and the resulting equations in a_n and c_n were then solved using Gaussian elimination. With the chosen heat flux the perturbation bulk temperature is zero. Accordingly, the wall to bulk temperature difference at each point may be determined, which when added to that for the uniform flux component gives the complete solution for the temperature and hence heat transfer coefficient.

The results are displayed in Fig. 3(b) using polar coordinates. This graphical representation convincingly demonstrates the points made by Reynolds [3], concerning the influence of the variation of heat flux. The numerical values are in excellent agreement. Typically, Reynolds [3], gives the analytical result:

$$Nu = (1+\cos\alpha) \bigg/ \left(\frac{11}{48}+\frac{\cos\alpha}{2}\right) \quad (11)$$

which confirms the values shown in Fig. 3(b) at $\alpha = 0$

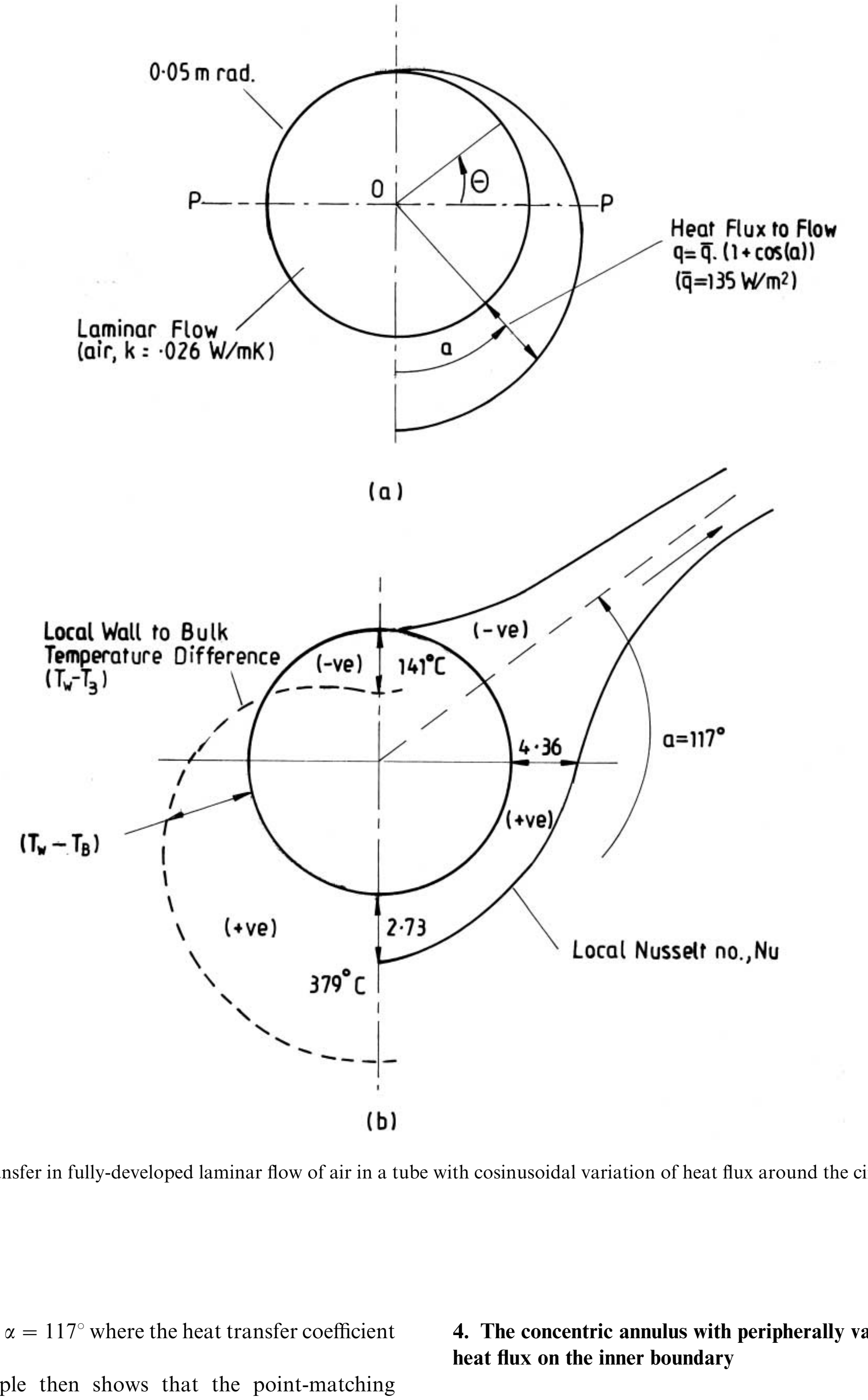


Fig. 3. Heat transfer in fully-developed laminar flow of air in a tube with cosinusoidal variation of heat flux around the circumference.

and $(\pi/2)$ and $\alpha = 117^\circ$ where the heat transfer coefficient is infinite.

This example then shows that the point-matching method leads to reliable results in this case, and that it may possibly be used in the more general case of the concentric annulus.

4. The concentric annulus with peripherally varying heat flux on the inner boundary

The solution procedure for this particular problem has been described in detail in a previous section. The example chosen is shown in Fig. 4 and refers to an internally

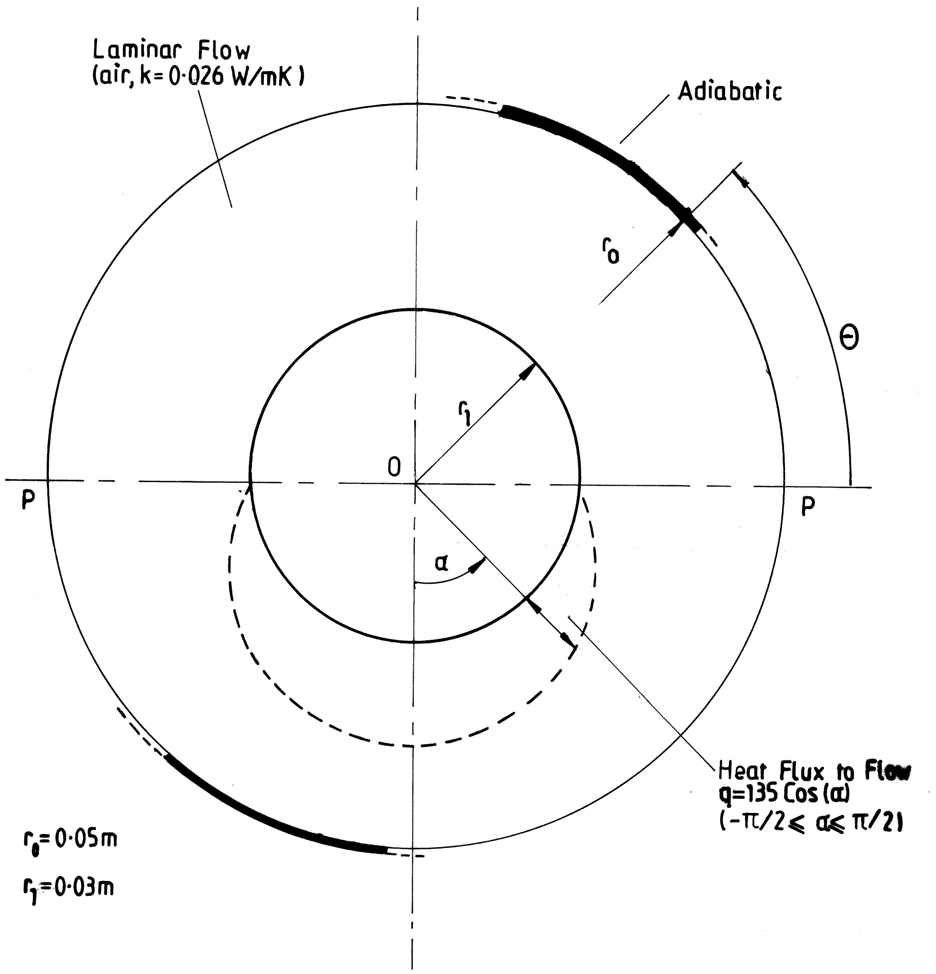


Fig. 4. The internally heated concentric annular duct with peripherally varying heat flux on the inner boundary.

heated annulus of the same geometry used earlier but now with a heat flux on the inner surface varying as $q_1 = 135\cos(\theta+\pi/2)$ for $(-\pi \leqslant \theta \leqslant 0)$, the outer surface being adiabatic. As in the tube example, the heat flux is disposed in such a way that the general form of equation (4) is appropriate in dealing with the perturbation components, the temperature differences are obtained using the fundamental solution of the second kind (7)–(10) and the point-matching solution, respectively. The local inner-wall to bulk temperature differences are then finally added and local heat transfer coefficients on the inner wall determined in the usual manner.

Of course, points on both boundaries of the annulus are used in the point-matching calculation to determine the perturbation temperature field, which interestingly, corresponds to that of thermal conduction in the cross-section with heat flowing in and out over parts of the inner boundary only.

The results are displayed in Fig. 5, using Cartesian coordinates in this case. Figure 5(a) shows how the total inner-boundary heat flux is apportioned into a uniform component and a perturbation component which of course has a line integral equal to zero. The temperatures are displayed in Fig. 5(b), that for the uniform heat flux being a constant obtained from the fundamental solution for axisymmetric heat transfer given in Table 8.1, ref. [12]. The total wall to fluid temperature difference is simply the summation of ΔT_{pB} and ΔT_{uB}. Finally, the distribution of heat transfer coefficient is plotted in Fig. 5(c) which includes the Nusselt number for a uniform heat flux on the inner boundary and an insulated outer wall.

The infinite heat transfer coefficient is reached at $\alpha = 86.5^\circ$, i.e. where the total temperature difference is zero. Beyond $\alpha = 90^\circ$, the heat transfer coefficient is necessarily zero.

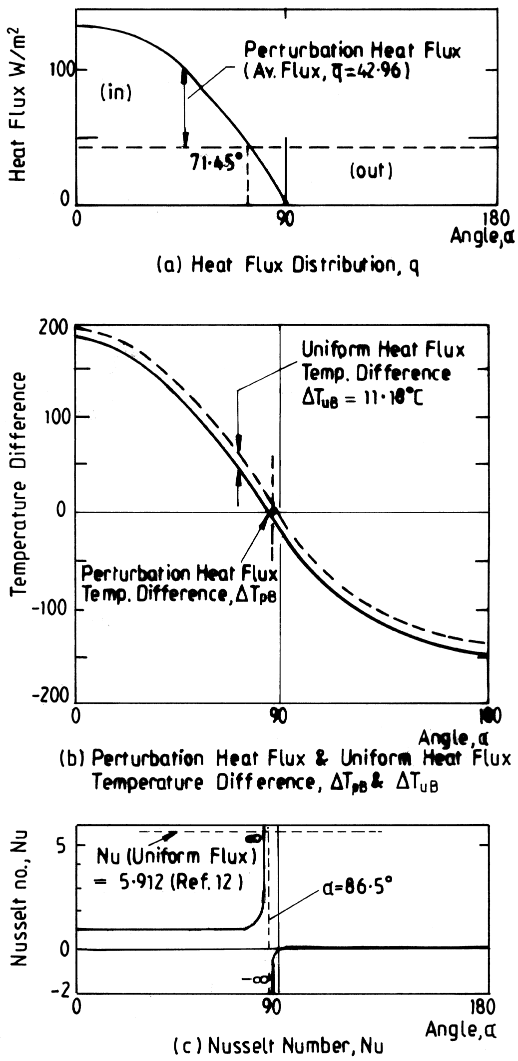


Fig. 5. The results for the annulus problem.

These data refer to the calculation made with 13 points, (and hence 13 unknowns), around the boundary of the annulus. Other numbers of points were tested but it was found that 13 gave an average value of $(T_{w1} - C_3)$ closest to zero. Of course there are unlimited combinations of annular geometry and boundary heat flux distributions, and a sequence of calculations in line with the axi-symmetric case with various ratios of inner and outer boundary heat fluxes (see ref. [12]) is not possible. The present example does however serve to illustrate the calculation procedure for the annular geometry, and the need to take cognizance of asymmetry of flux on wall temperature levels in such geometries.

Albeit that the chosen annulus and tube examples are in no way directly comparable, it is to be noted that the variation of *Nu* (apart from the infinite value) in the annular geometry is significantly larger.

5. Average heat transfer coefficient

The heat transfer performance of a surface is frequently measured in terms of an 'average heat-transfer coefficient'. In the present investigation, where the local heat transfer coefficient, *h*, varies peripherally, the average value may be defined in two ways. As pointed out in ref. [16], the mathematical average value is the arithmetic mean of the local values while a pseudo average value based on the quotient of the mean heat flux and the mean temperature difference is sometimes used. In general these two values are different, the magnitude of the difference depending on the peripheral flux and temperature gradients. Accordingly, it is incorrect to compare average heat transfer coefficients obtained experimentally using average surface heat flux and average temperature with mathematical mean values obtained analytically.

In the present examples where this is an infinite value of the heat transfer coefficient, it is not possible to calculate the mathematical average numerically.

However, in the simple case shown in Fig. 2, it is easy to show that the mathematical average of *h* over the whole boundary of the annulus is different from the pseudo-average based on average flux and temperature. Accordingly, caution must be exercised whenever calculations and comparisons of average heat transfer coefficients are being made. The ratio of these two averaged heat transfer coefficients may be greater or less than unity depending on the flux and temperature gradients in the boundary wall, this being a further illustration of the conjugate nature of the heat transfers.

6. Discussion and conclusions

A theoretical investigation of heat transfer in fully-developed laminar flow in an annulus with peripherally varying heat flux has been carried out. The calculation involves the superposition of the solutions for an axi-symmetric heat flux and a perturbation heat flux both of which are constant axially. The former solution is readily available in the literature, while that for the perturbation heat flux is obtained using a numerical point-matching method.

In order to test the present procedure, the example of a tube with cosinusoidally varying heat flux (see ref. [3]), has been calculated and found to give results in exact agreement with those obtained by Reynolds [3]. Accordingly, the calculation procedure was then extended to an annular flow which is heated internally by a peripherally varying flux on the inner wall. As for the tube, a similar pattern in the results for both temperature and heat transfer coefficient has been obtained.

A by-product of the calculation is the two-dimensional temperature field associated with the perturbation heat flux. This of course corresponds to the heat conduction solution in the annular cross-section arising out of a heat flow into the annulus and an equal heat flow out of the annulus over the inner boundary, as shown in Fig. 5(a). A separate plot of the perturbation isotherms is found to be in accord with these heat flows over the cross-section, lending further confidence to the results of the point-matching procedure. Optimization of the number of points chosen may be effected by repetitive calculations. The spacing between points may of course be variable and it would appear prudent to select finer discretization in the vicinity of discontinuities of flux such as that at $\alpha = 90^\circ$ (or $\theta = 0$). Other calculations using heat flux variations with more severe discontinuities caused some difficulties, the case of a constant flux over a part of the boundary being a typical example. Such a case is however unrealistic in practice and this matter was not pursued any further. Another test of the accuracy of the prediction of the perturbation temperature is symmetry about the appropriate axis of the cross-section and this test was applied in all cases by examining the relative magnitudes of the temperatures at the corresponding points.

A distinctive feature of the heat transfer coefficient distribution in the examples studied, is the infinite magnitude of the Nusselt number at a particular angular location. This somewhat curtails the investigation on the 'average heat transfer coefficient' since the 'mathematical average value' here cannot be determined analytically from a set of discrete numerical values. However, as explained earlier, caution has to be exercised when comparing arithmetic mean values with experimentally determined average values based on average flux and average temperature difference, since these are not the same quantities.

The existence of the infinite heat transfer coefficient is little cause for concern: in fact, there are numerous situations where this may occur and so the present case is in no way exceptional in this respect. More important is the influence of variation of flux on the local wall temperature, the disposition of which may be the critical factor in the engineering situation. The present method enables these wall temperatures to be predicted with considerable confidence.

While one of the main purposes of the present enquiry has been to extend the annulus problem to include such effects, it is clear that, in principle, the procedure may be used for any cross-section duct with any peripheral flux variation. Again, however, the wide diversity of geometries and associated flux distributions in any particular shape makes a parametric study of this particular problem an impossible task.

For the anulus however, part of the solution is at hand in the tabulated list of fundamental solutions of the second kind, and it remains to determine the remaining perturbation solution for the particular case by the point-matching numerical procedure.

Appendix: the perturbation bulk temperature, T_{pB}

As indicated in the second section, the bulk temperature associated with the perturbation heat flux is calculated using equation (3) with the appropriate substitutions for the velocity u and the temperature $\hat{T}$. In the general case this may be effected numerically: however, an elegant method used earlier by Savino et al. [15], for a rectangular duct, may be adapted for the present annular geometry. Savino et al. converted the equation for the bulk temperature using the energy equation and then used the second form of the Green's theorem to develop an expression for the bulk temperature in terms of a line integral. In the present case of the annulus, the same procedure may be followed taking cognizance of the fact that an annulus is a doubly-connected duct in the line integral. Fortuitously, in the present formulation the bulk temperature for the perturbation heat flux component is zero (apart from the added constant, C_3). This is because the complementary uniform heat flux component is axisymmetric. Accordingly, integration of equation (3) is avoided in the annulus (and pipe) problem and the wall to bulk temperature difference for the perturbation flux is simply:

$$(T_{pw}-T_{pB}) = (T_{pw}-C_3)-(T_{pB}-C_3)$$

$$(T_{pw}-T_{pB}) = b_0 \ln r_w + \sum_{n=1,2\ldots} (a_n r_w^n + b_n r_w^{-n}) \cos(n\theta) + \sum_{n=1,2\ldots} (c_n r_w^n + d_n r_w^{-n}) \sin(n\theta). \quad \text{(A1)}$$

Once the constants b_0, a_n, b_n, c_n and d_n have been determined by the point-matching method, the local heat transfer rate may be calculated.

Furthermore, it is interesting to consider the case when the perturbation heat flux is symmetrical about the diameter as shown in Fig. 1 and when (in addition) the perturbation heat flux is an odd function about $\theta = \pi/2$. In this case, it can be shown [14], that

$$\hat{T}(r,\theta) = -\hat{T}(r,\pi-\theta) \quad \text{(A2)}$$

when again, from equation (3), the perturbation bulk temperature

$$T_{pB}(-C_3) = 0. \quad \text{(A3)}$$

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